Frictional drag in dilute bilayer 2D hole systems

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We develop a theory for frictional drag between two 2D hole layers in a dilute bilayer GaAs hole system, including effects of hole-hole and hole-phonon interactions. Our calculations suggest significant enhancement of hole drag transresistivity over the corresponding electron drag results. This enhancement originates from the exchange induced renormalization of the single layer compressibility and the strong dependence of single layer conductivity on density. We also address the effect of hole-phonon interaction on the drag temperature dependence. Our calculated results are in reasonable quantitative agreement with recent experimental observations.
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Frictional drag measurements of transresistivity in modulation doped GaAs electron bilayer systems have led to significant advances in our understanding of density and temperature dependence of electron-electron and electron-phonon interactions in 2D systems [1]. Recent interest [2–5] has focussed on the role of electron correlation effects on the drag resistivity, which should vary in a systematic manner as a function of electron density and temperature. In particular a recent experiment by Pillarisetty *et al.* [6] reports drag measurements in very low density and high quality hole bilayers, where Coulomb interaction (i.e., correlation) effects should be strong by virtue of the large GaAs hole effective mass ($m_h^* \approx 0.4m$ for holes compared with $m_e^* \approx 0.07m$ for electrons where m is the vacuum electron mass) and very low hole density (hole density $p = 10^{10} - 10^{11} cm^{-2}$ in Ref. [6], whereas typical electron densities used bilayer drag measurements have been comparatively high, $> 10^{11} cm^{-2}$). In terms of the dimensionless interaction strength parameter $r_s \equiv (p\pi)^{-1/2} m^* e^2/(\hbar^2 \kappa)$ where κ is the background dielectric constant, which measures the ratio of the potential energy to the kinetic energy in the interacting hole system, Ref. [6] explores the strongly correlated regime of $r_s \approx 20-40$ whereas the earlier electron drag experiments explored the weak coupling regime of $r_s < 3$.

The experimental findings of Ref. [6] were striking, and motivated our work. They are: *a.* the low density hole drag is $2-3$ orders of magnitude larger than the corresponding electron drag results published in the literature [1–3,7]; *b.* There are some small (but systematic) deviations of the observed low density hole drag resistivity from the expected $\rho \sim T^2$ Fermi liquid behavior. The low density data of Ref. [6] seem to better fit a $T^{2.5}$ behavior at low temperatures; *c.* The observed ρ/T^2 behavior in Ref. [6], plotted as a function of T for various bilayer hole densities, is qualitatively similar to the corresponding electron drag results in the sense that ρ/T^2 at a fixed density shows a peak at some temperature T_p which decreases with decreasing density – the peak in ρ/T^2 as a function of T at the lowest hole density $p = 10^{10} cm^{-2}$ is very sharp; *d.* For bilayers with unequal hole densities, the drag resistivity at a fixed temperature plotted as a function of the density ratio p_1/p_2 decreases monotonically and does not exhibit a peak at the balance point as it does in the corresponding electron case. This peak is believed to arise from the $2k_F$ phonon scattering.

These experimental findings become particularly interesting due to their possible relation to the collection of transport anomalies in 2D systems referred to as the 2D metal-insulator transition (2D MIT) phenomena (The samples of Ref. [6] exhibit 2D MIT in each layer at $p \approx 8.5 \times 10^9 cm^{-2}$). These anomalies, observed largely in the large r_s regime, have raised doubts regarding the applicability of Fermi liquid theory to two dimensional systems of charges (electrons or holes) at the large r_s regime, and this question has been widely debated in the literature (see [8] for a review). In that context, then, it is particularly important to examine whether drag measurements in the large r_s regime may be understood within the Fermi liquid framework.

In this paper we attempt, and largely succeed, to interpret the hole drag data of Ref. [6] within a Fermi liquid approach. The important inputs to our theory are a perturbative expression to the drag resistivity, based on a perturbative treatement of inter-layer hole-hole scattering rate, a Hubbard approximation for the single layer polarization operator, and the experimentally measured density dependence of the single layer conductivity. Inputs of more minor significance are form factors that account for the finite thickness of the two layers and hole-phonon interaction parameters.

Our starting point for the calculation of the drag resistivity is the following expression:

$$\rho = \frac{\beta}{\sigma_1 \sigma_2} \frac{d\sigma_1}{dp_1} \frac{d\sigma_2}{dp_2} \int \frac{q^2 d^2 q}{(2\pi)^2} \frac{d\omega}{2\pi} \frac{F_1(q,\omega) F_2(q,\omega)}{\sinh^2(\beta\omega/2)}, \quad (1)$$

where $F_i(q,\omega) = \mathrm{Im}\Pi_{ii}(q,\omega)|u_{12}^{sc}(q,\omega)|$. In Eq. (1),

$\beta = 1/T$ is the inverse temperature (we use units such that $k_B = \hbar = 2e = 1$, except in final formulas); $p_{1,2}$ are the hole densities in layers 1 and 2; $\sigma_{1,2}$ are the conductivities of each layer; q is the 2D wave vector in the layer; Π_{11}/Π_{22} are the irreducible hole polarizabilities in each layer; and u_{12}^{sc} is the dynamically screened effective interlayer interaction. Several comments are in place regarding this expression: first, when the drag resistivity is derived from the Boltzmann equation ([9,10], and see also [11]), the approximation $\frac{d\sigma_i}{dp_i} \approx \frac{\sigma_i}{p_i}$ is being made. This approximation is valid for well conducting layers, but becomes invalid at the low densities relevant here (see below). The need to replace $\frac{\sigma_i}{p_i}$ by $\frac{d\sigma_i}{dp_i}$ is discussed in Ref. [12]. Second, the dynamically screened interlayer interaction u_{12}^{sc} satisfies a matrix Dyson equation,

$$\mathbf{u}(q,\omega) = \left[\mathbf{1} - \mathbf{v}_t(q,\omega)\Pi(q,\omega)\right]^{-1} \mathbf{v}_t(q,\omega). \qquad (2)$$

We include in the bare interaction $\mathbf{v}_t(q,\omega) = \mathbf{v}^c(q) + \mathbf{v}^{ph}(q,\omega)$ both the hole-hole direct Coulomb interaction and the phonon mediated interaction $\mathbf{v}^{ph}$, which includes the acoustic phonon propagator and the appropriate hole-phonon interaction matrix element [13]. The phonon mediated interaction includes both deformation potential and piezoelectric couplings between the holes and the acoustic phonons using the standard hole-phonon interaction parameters for GaAs [10]. Following the common approximation we assume the inter-layer polarization operators Π_{12}, Π_{21} to be zero.

When Eq. (1) is used to analyze experiments carried out on identical layers in the commonly explored regime, of high density and low temperature, it yields,

$$\rho_D = \frac{\zeta(3)\pi}{32}\frac{h}{e^2}\left(\frac{k_B T\kappa}{e^2 k_F^3 d^2}\right)^2 \qquad (3)$$

with ζ being the Riemann zeta function. The assumptions involved in getting from Eq. (1) to Eq. (3) are large inter-layer separation ($k_F d \gg 1$, $q_{TF} d \gg 1$, with k_F being the Fermi wave vector and q_{TF} being the Thomas Fermi screening wave vector), Drude relation between conductivity and density ($\sigma_i \propto p_i$), and a Random Phase Approximation (RPA) in which Π_{ii} is replaced by its value for non-interacting electrons. The contribution of electron-phonon interaction is negligible in this regime. Eq. (3) is typically smaller than the experimentally measured value, by a factor of $2-5$. For the experiment of Ref. [6], the disagreement is much larger, getting as large as a factor of 500.

Our calculation differs from that leading to Eq. (3) in five points, all of them leading to an increase of the drag resistivity.

First, we use the Hubbard approximation to calculate the polarization operators Π_{ii}. Within this approximation,

$$\Pi_{ii}(q,\omega) = \left[1 + v_i^c(q)\Pi_{ii}^0(q,\omega)G(q)\right]^{-1}\Pi_{ii}^0(q,\omega), \qquad (4)$$

where $\Pi_{ii}^0(q,\omega)$ is the polarization operator for non-interacting electrons, and $G(q) = q/2\sqrt{q^2+k_F^2}$ is the local field correction. For small q the Hubbard approximation amounts to the introduction of a Landau parameter $f_0 = -v_i^c(q)G(q) \approx \frac{\pi e^2}{k_F}$, making the inverse compressibility of each layer $\frac{\partial\mu}{\partial n} = \frac{2\pi\hbar^2}{m} - \frac{\pi e^2}{k_F}$. This approximation is then consistent with measurements of the compressibility that yield negative values. In general, the effect of the Hubbard approximation is to decrease the bare intralayer interaction by a factor of $1 - G(q)$. The fact that the exchange-driven local field correction, $G(q)$, affects only the intra-layer interaction is what makes its effect significant: the interaction potential between anti-symmetric charge densities, $v_{11}^c(q)(1-G(q)) - v_{12}^c(q)$, is modified from $2\pi e^2 d/\kappa$ to $\frac{2\pi e^2}{\kappa}(d - \frac{1}{2k_F})$. This increases the drag resistivity, and in the limit of $k_F d \gg 1$, $q_{TF} d \gg 1$, Eq. (3) is replaced by

$$\rho_D = \frac{\zeta(3)\pi}{32}\frac{h}{e^2}\left(\frac{k_B T\kappa}{e^2 k_F^3 d^2}\right)^2\left(\frac{2k_F d}{2k_F d - 1}\right)^4. \qquad (5)$$

We note by passing that even for densities that are not very low, the fourth power of the last term in Eq. (5) makes the correction quite significant. In the low density regime we consider, the approximation Eq. (5) is not valid, since scattreing events with large momentum transfer have a significant contribution. Our calculations below show that in that regime the local field correction increases the drag resistivity by about an order of magnitude.

Second, we do not use the approximation $\sigma_i \propto p_i$. Rather, we use measured values of $\sigma(p)$, provided to us by Pillarisetty *et al.* [14], to extract $\frac{d\sigma_i}{dp_i}$ for use in our calculation of the drag resistivity. At the lowest density measured $\left(\frac{d\sigma_i}{dp_i}\frac{p_i}{\sigma_i}\right)^2 \simeq 10-20$, so this correction increases the drag resistivity by about an order of magnitude. These measurements of $\sigma(p)$ are limited, at this stage, to the range of densities between $0.72 - 2.15 \cdot 10^{10} cm^{-2}$, and to the temperature range $T < 0.5K$. While this correction deacreases with increasing density and temperature, it is still between $3-5$ at the edge of the measured range. Our calculation for the range where experimental values of $\left(\frac{d\sigma_i}{dp_i}\frac{p_i}{\sigma_i}\right)^2$ are available is presented in Fig. 1. The other figures do not include this factor, for lack of availability of experimental data.

Third, the assumption $k_F d >> 1$, implying that the relevant momentum exchange is much smaller than k_F, is not valid at the low density regime. Consequently, there is an important contribution due to large momentum transfer, $\hbar q \approx 2\hbar k_F$ scattering. Taking this contribution into account (by a numerical integration of Eq. (1)) leads to an increase of the drag by another factor of ~ 2 as compared to the Boltzmann result.

Fourth, the introduction of finite thickness form factors [15] (see to the Coulomb and phonon interaction)

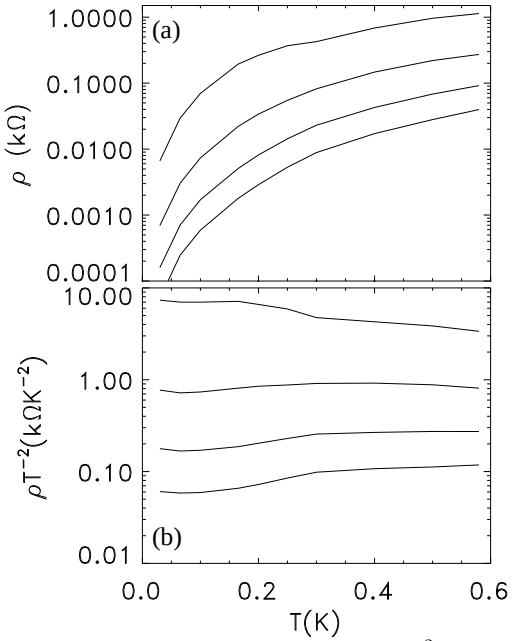

FIG. 1. The drag resistivity (a) and $\rho(T)/T^2$ (b) as a function of temperature for various hole densities (p = 1.0, 1.5, 2.0, 2.5 $\times 10^{10} cm^{-2}$, from top to bottom) calculated with all five correction factors as explained in the text. Throughout this paper we use a hole bilayer system with the layer separation of $d = 300\mathring{A}$ and the well width of $a = 150\mathring{A}$.

effectively decreases d, and increases the drag resistivity by about a factor 2.

And fifth, we include the phonon contribution to the interaction in addition to the Coulomb one. This changes the result by less then 50%.

Combining all these five factors, we are able to account for most of the results of the measurements. We account for the very large increase of drag, as compared to measurements of electronic systems. Our Fig. 1 is in good quantitative agreement with the measured data, to within a factor of 2. This type of agreement is similar to what is obtained in the small r_s limit. Our analysis yields a leading quadratic temperature dependence of the drag in the limit $T \longrightarrow 0$ (at least as long as the conductivity σ is temperature independent in that limit). However, our numerical integration of Eq. 1, as presented in Fig. 2, indicates that even at the lowest measured temperatures the drag resistivity does not follow a T^2 dependence. There are a number of reasons for that. First, at the low densities used in Ref. [6] [shown as the inset in our Fig. 2 to be compared with inset (b) in Fig. 3 of Ref. [6]] the Fermi energy $(1 - 2K)$ is not much larger than the measurement temperature range. Second, even well below $1K$ the phonon contribution to drag in the low density hole bilayers is quite substantial (in contrast to electron systems where the phonon contribution is typically a factor of 10^3 smaller for small layer separations of $d = 300\mathring{A}$ or so used in Ref. [6]). As such we believe that the experimental departure from the T^2 behavior

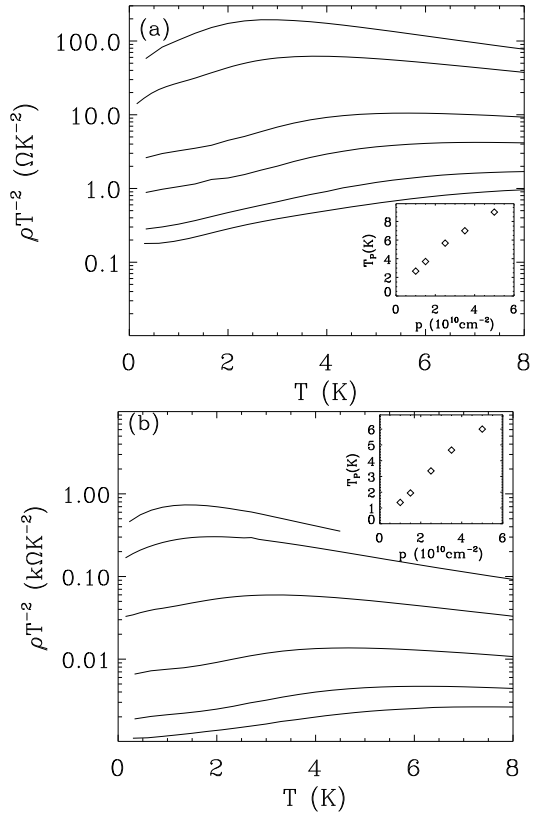

FIG. 2. The calculated total hole frictional drag $\rho(T)/T^2$ as a function of T for various hole densities (p = 1.0, 1.5, 2.5, 3.5, 5.0, 7.0 $\times 10^{10} cm^{-2}$, from top to bottom) within (a) RPA dynamical screening theory, and (b) HA. In insets we show the variation in the calculated peak temperature T_p as a function of the hole density p.

reported in Ref. [6] is essentially a manifestation of the fact that phonon effects remain significant in the experiments, and the asymptotic T^2 regime is hard to reach in hole systems. We find that our calculated $\rho(T)$ at low temperature is well approximated by a $T^{2.4}$ behavior for $p = 2.0 \times 10^{10} cm^{-2}$ and the exponent increases as the hole density decreases. In Fig. 4 we show our calculated contributions to the hole drag resistivity from individual hole-hole and hole-phonon interactions as compared with the corresponding electron case. The importance of phonon effects to the hole drag transresistivity is manifestly evident in Fig. 4.

In Fig. 3 we qualitatively "explain" the particularly anomalous feature of the data in Ref. [6] [shown as the inset in our Fig. 2 to be compared with inset (b) in Fig. 3 of Ref. [6]], i.e., the non-existence of a peak in ρ/T^2 as a function of the density ratio p_1/p_2. In qualitative agreement with Ref. [6] the calculated drag resistivity at a fixed temperature decreases monotonically as a function of the hole density ratio p_1/p_2 without showing any phonon-induced peak at the balance point $p_1 = p_2$, as has been observed in corresponding electron bilayer experiments [7]. This peak arises from the sharp 2D Fermi surfaces in the two electron layers which, when perfectly

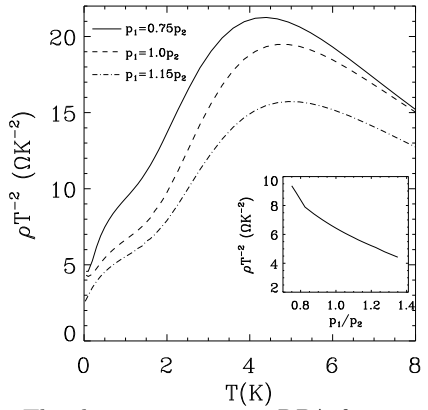

FIG. 3. The drag resistivity in RPA for various density ratio (p_1/p_2, where $p_1 = p_{drive}$ and $p_2 = p_{drag}$) with the fixed drag layer density ($p_2 = 2.0\times10^{10}cm^{-2}$). Inset shows the drag resistivity as a function of a density ratio at $T = 1.0K$.

matched at the balance point $p_1 = p_2$, leads to enhanced phonon scattering, leading to the peak resistivity at $p_1 = p_2$. No such peak exists in the hole bilayer case because of the small Fermi temperature in the hole case, $T_F = 1.4K$, and the large Bloch-Grüneisen temperature, $T_{BG} = 2.8K$, (for $p = 2\times10^{10}cm^{-2}$). Therefore typical T/T_F is rather large in the hole case, leading to thermally broadened Fermi surfaces in the low density hole bilayers of Ref. [6] which cannot exhibit any sharp Fermi surface effects, and consequently the so-called "$2k_F$ phonon peak" [7] arising from the Fermi surface matching is absent. The peak in ρ_D/T^2, appearing in Fig. 3, takes place at high temperatures, above the Fermi energy, and is unlikely to be related to the experimental peak that appears around $0.5K$. We believe it may be related to the two opposing temperature dependences of the hole-hole

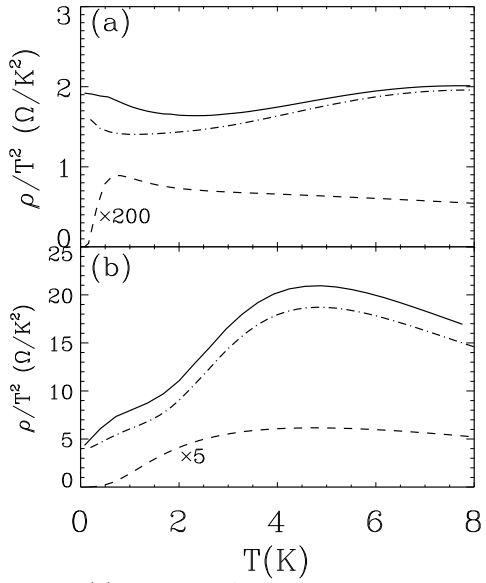

FIG. 4. (a) The RPA-calculated electron drag resistivities from electron-electron v^c (dot-dashed), electron-phonon v^{ph} (dashed), and total interaction v_t (solid) for $n = 2.0\times10^{10}cm^{-2}$. (b) The same for the hole drag resistivity.

scattering rate on one hand and the $\left(\frac{d\sigma_i}{dp_i}\frac{p_i}{\sigma_i}\right)^2$ factor on the other hand.

In conclusion we have developed a theory for frictional interlayer drag resistivity in low density hole bilayers including both hole-hole and hole-phonon interaction effects. We explain the observed [6] dramatic deviation of drag resistivity from an RPA-based Boltzmann equation calculation as arising from several factors, out of which the most important ones are the dependence of the single-layer resistivity on density and the exchange induced renormalization of the single-layer polarization operator. Our calculation explains the large increase in the magnitude of drag and the absence of phonon-induced $2k_F$ peak in the drag between matched densities. We also obtain reasonable qualitative understaning of the temperature dependence of the drag resistivity. Our calculation are all done within a perturbative Fermi-liquid based approach.

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